



## Power of Frobenius Endomorphism and its Performance on PseudoTNAF System

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### Abstract

Let  $E$  be an elliptical curve defined over  $F_{2^m}$  and the mapping  $\tau$  is a Frobenius endomorphism from the set  $F_{2^m}$  to itself. The Koblitz curve is a special curve whose  $\tau$  has been used to improve the calculation performance of its scalar multiplication,  $nP$  where  $P$  is a point on the curve  $E$ . Moreover, the multiplier,  $n$  is  $\tau$ -adic non adjacent form (TNAF) expansion where its digit is generated by the repeated division of an integer in the ring of  $Z(\tau)$  by  $\tau$ . Previous research has found that the power of Frobenius endomorphism  $\tau^m$  has some advantages in TNAF, Reduced TNAF and their equivalent i.e. pseudoTNAF expansions. In this paper, new finding of  $\tau^m$  based on  $v$ -simplex and arithmetic sequences is provided. With this approach, the performance of converting modulo  $\rho \frac{\tau^m - 1}{\tau - 1}$  to  $r + s\tau$  an element of  $Z(\tau)$  in pseudoTNAF's system is enhanced.

**Keywords:** cryptography; field; Frobenius endomorphism; Koblitz curve; number of elliptic points; sequence of arithmetic; sequence of simplex;  $\tau$ -adic non adjacent.